Estimating the Cost-Effective Checking Request Policies

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Abstract

Suppose that an inspector is requested to check the condition of a failure-prone machine and arrives with any time delay at the facility where the machine is equipped. Since the failure of machine can be detected only by checking, the problem is to determine the checking request time maximizing the criterion called the *cost effectiveness*. In this paper, we consider two checking request policies maximizing the cost effectiveness criteria, and develop the statistical algorithms to estimate them, provided that the complete failure time data are available.

1 Introduction

Consider a failure-prone machine whose failure can be detected only by checking. Without any loss of generality, it is assumed that the checking is perfect, *i.e.* the machine does not deteriorate/fail by checking. Suppose that the inspector is requested to check the condition of a machine at any time after it begins operating, and that the constant time period called the delay time or waiting time is needed for the inspector to arrive at the facility where the machine is equipped. Checking is carried out as soon as the inspector arrives there and the time for checking itself can be negligible. If the machine has already failed up to the inspector's arrival time point, the corrective replacement of the failed machine is made at that time of point. On the other hand, *i.e.*, if the machine does not fail during the checking request period and/or the waiting time period, we can consider two cases: In Model 1, the preventive replacement is performed immediately at the arrival time point. In Model 2, the inspector continues monitoring the condition of the machine and waits until it fails.

Yamada and Osaki (1980, 1981) consider these two models and derive the optimal checking request policies maximizing the cost effectiveness criteria, which are proposed by Trott (1965) and Winlund (1965). More precisely, Yamada and Osaki (1980, 1981) describe the stochastic behavior of the above models by Markov renewal processes and obtain analytically not only the steady-state availability and the expected number of visits to arbitrary states per unit time in the steady state but also the optimal checking request policies. Yamada and Osaki (1978) further extend the above models and propose the different checking request policy taking account of the surveillance limit.

In this paper, we consider the same checking request policies as Yamada and Osaki (1980, 1981), and develop the statistical algorithms to estimate the optimal checking request policies which maximize the cost effectiveness, provided that the complete failure time data are available. By applying the similar but somewhat different technique based on the equilibrium distribution from Aven (1987) and Dohi *et al.* (1996), we provide non-parametric estimators of the optimal checking request policies.

2 Model Description

Notation:

We define the following notation: $F(t)$: continuous lifetime distribution, λ (> 0): MTTF (Mean Time To Failure), $\psi(\cdot) = 1 - \psi(\cdot)$: survivor function, L (> 0): delay (waiting) time, t_0 (≥ 0): checking request time (decision variable), c_c (> 0): checking cost, c_r (> 0): corrective or preventive replacement cost, c_d (> 0):

system down cost per unit time, c_s (> 0): surveillance cost per unit time, $C_j(t_0)$: expected cost per unit time in the steady state for Model j ($= 1, 2$), $A_j(t_0)$: steady-state system availability for Model j ($= 1, 2$), $E_j(t_0)$: cost effectiveness for Model j ($= 1, 2$).

Model 1:

Let us consider the situation where a machine operation starts at time $t = 0$, where the machine failure may be identified only by checking. Suppose that the checking is requested at time t_0 just after instruction of the original machine. The inspector arrives at the facility where the machine is equipped, after the delay time L which is assumed to be constant, and checks the condition of the machine. If the machine has already failed, then it is replaced correctively by a new one, otherwise the preventive replacement is performed, where the lifetime distribution of the machine has an arbitrary and absolutely continuous probability distribution function $F(t)$ with finite mean λ. Then, from the familiar renewal reward argument, we obtain the expected cost per unit time in the steady state and the steady-state system availability as

$$\begin{aligned} C_1(t_0) &= \Big\{c_c + c_r + c_d \int_0^{t_0+L} \overline{F}(x)dx\Big\}/(L + t_0), \qquad (1) \\ A_1(t_0) &= \Big\{\int_0^{t_0+L} \overline{F}(x)dx\Big\}/(L + t_0), \qquad (2) \end{aligned}$$

respectively.

Model 2:

In Model 1, it is assumed that the inspector replaces the machine even if it is still operating when he/she arrives at the facility. However, since the inspector can identify the machine failure when it happens, the preventive replacement of the non-failed machine may not be always useful. That is, if the inspector can monitor the condition of the machine continuously until it fails, the corrective replacement of the failed machine is carried out at the failure time point. This is due to the assumption that the preventive replacement cost is equivalent to the corrective maintenance one. In this situation, the expected cost per unit time in the steady state and the steady-state system availability are given by

$$\begin{aligned} C_2(t_0) &= \Big\{c_c + c_r + c_d \int_0^{t_0+L} \overline{F}(x)dx + c_s \int_{t_0+L}^{\infty} \overline{F}(x)dx\Big\}/\Big\{\lambda + \int_0^{t_0+L} F(x)dx\Big\}, \qquad (3) \\ A_2(t_0) &= \lambda/\Big\{\lambda + \int_0^{t_0+L} F(x)dx\Big\}, \qquad (4) \end{aligned}$$

respectively.

3 Optimal Checking Request Policies

Following Yamada and Osaki (1980, 1981), define the cost effectiveness by

$$E_j(t_0) = \frac{[\text{steady} - \text{state system availability}]}{[\text{expected cost per unit time in the steady state}]} = \frac{A_j(t_0)}{C_j(t_0)}, \qquad j = 1, 2, \qquad (5)$$

which denotes the mean operative time per unit mean cost. Yamada and Osaki (1980, 1981) derive the optimal checking request policy t_0^* which maximizes the cost effectiveness $E_j(t_0)$ ($j = 1, 2$). The following results summarize the optimal checking request policies under the cost effectiveness criteria.

Proposition 3.1: (i) For Model 1, if $(c_r + c_c)\overline{F}(L) > c_d \int_0^L x dF(x)$, then there exists a finite and unique optimal checking request policy t_0^* ($0 < t_0^* < \infty$) satisfying the non-linear equation:

$$\{c_r + c_c + c_d(t_0^* + L)\}\overline{F}(t_0^* + L) = c_d \int_0^{t_0+L} \overline{F}(x)dx \qquad (6)$$

with

$$E_1(t_0^*) = \overline{F}(t_0^* + L)/\{c_d F(t_0^* + L)\}, \tag{7}$$

otherwise $t_0^* = 0$ with $E_1(0) = \int_0^L \overline{F}(x)dx/\{c_r + c_c + c_d \int_0^L F(x)dx\}$, *i.e.* it is optimal to request the inspection when the machine begins operating.

(ii) For Model 2, if $c_s > (c_d + c_s)F(L)$, then there exists a finite and unique optimal checking request policy t_0^* $(0 < t_0^* < \infty)$ satisfying the non-linear equation:

$$(c_s + c_d)F(t_0 + L) = c_s \tag{8}$$

with

$$E_2(t_0^*) = \lambda/\Big\{c_r + c_c + c_s\lambda - (c_d + c_s)\int_0^{t_0^*+L} xdF(x)\Big\}, \tag{9}$$

otherwise $t_0^* = 0$ with $E_2(0) = \lambda/\{c_r + c_c + c_d \int_0^L F(x)dx + c_s \int_L^\infty \overline{F}(x)dx\}$.

In the following section, we consider the situation where the lifetime distribution is unknown but the corresponding lifetime data are available.

4 Non-Parametric Estimation of Optimal Policies

Define the equilibrium distribution of the lifetime distribution function:

$$F^*(t) = \frac{1}{\lambda}\int_0^t \overline{F}(x)dx. \tag{10}$$

It is seen immediately that $F^*(0) = 0$ and $F^*(\infty) = 1$. The following result gives a graphical interpretation for Proposition 3.1.

Theorem 4.1: (i) For Model 1, the optimal checking request policy has to satisfy

$$\max_{0 \le t_0 < \infty} : \frac{F^*(t_0 + L)}{(c_c + c_r)/c_d + t_0 + L}. \tag{11}$$

(i) For Model 2, the optimal checking request policy has to satisfy

$$\max_{0 \le t_0 < \infty} : F^*(t_0 + L) - \frac{c_d}{\lambda(c_d + c_s)}(t_0 + L). \tag{12}$$

From Theorem 4.1, it can be found that the optimal checking request policy t_0^* for Model 1 is given by the point $t_0^* + L$ maximizing the tangent slope to the curve $F^*(t_0 + L)$ from the point $(-(c_c + c_r)/c_d, 0)$ in the two-dimensional plane $(t_0 + L, F^*(t_0 + L)) \in [L, \infty) \times [F^*(L), \infty)$. On the other hand, the optimal checking request policy for Model 2 can be characterized by the point $t_0^* + L$ with the maximum vertical distance from the straight line $c_d(t_0 + L)/\{\lambda(c_d + c_s)\}$ to the curve $F^*(t_0 + L)$. The conditions $(c_r + c_c)\overline{F}(L) > c_d \int_0^L xdF(x)$ and $c_s > (c_d + c_s)F(L)$ can be also interpreted geometrically on the graph.

Next consider the estimation problem of the optimal checking request policies. Suppose an ordered complete sample $0 = y_{(0)} \le y_{(1)} \le y_{(2)} \le \cdots \le y_{(n)}$ from the underlying distribution function F, which is unknown. We further define the data sequence $x_{(i)} = y_{(i)} + L$ $(i = 1, 2, \cdots, n; x_{(0)} = L)$. As a non-parametric estimator of the equilibrium distribution $F^*(t)$, we define the scaled total time on test statistics based on this sequence by

$$U_{in} = \frac{T_{in}}{T_{nn}}, \qquad i = 1, 2, \cdots, n, \tag{13}$$

where

$$T_{in} = \sum_{j=1}^{i} (n-j+1)(x_{(j)} - x_{(j-1)}), \qquad i = 1, 2, \cdots, n;\ T_{1n} = ny_{(1)}. \tag{14}$$

Plotting the point $(x_{(i)}, U_{in})$, $(i = 1, 2, \cdots, n)$ and connecting them by line segments yield a piecewise continuous curve in the two-dimensional plane. To this end, the optimization problem is reduced to find the point $x_{(i^*)}$ $(y_{(i^*)})$ so as to maximize the tangent slope to the curve $(x_{(i)}, U_{in})$ from the point $(-(c_c+c_r)/c_d, 0)$ or the vertical distance from the $c_d x_{(i)}/\{\lambda(c_d + c_s)\}$ to U_{in} $(i = 0, 1, \cdots, n)$.

The following theorem is the main result of this paper.

Theorem 4.2: Suppose that the optimal checking request policy for Model j $(= 1, 2)$ has to be estimated from an ordered complete sample $0 = y_{(0)} \le y_{(1)} \le y_{(2)} \le \cdots \le y_{(n)}$ of lifetime data from an absolutely continuous lifetime distribution F, which is unknown. For $x_{(i)} = y_{(i)} + L$ $(i = 1, 2, \cdots, n; x_{(0)} = L)$, the non-parametric estimators of the optimal checking request policies which maximize E_j are given by $\hat{t}_0^* = y_{(i^*)} = x_{(i^*)} - L$, where

$$i^* = \left\{ i \mid \max_{0 \le i \le n} : \frac{U_{in}}{(c_c + c_r)/c_d + x_{(i)}} \right\} \tag{15}$$

and

$$i^* = \left\{ i \mid \max_{0 \le i \le n} : U_{in} - \frac{c_d x_{(i)}}{\lambda(c_s + c_d)} \right\} \tag{16}$$

for Model j $(= 1, 2)$, respectively.

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